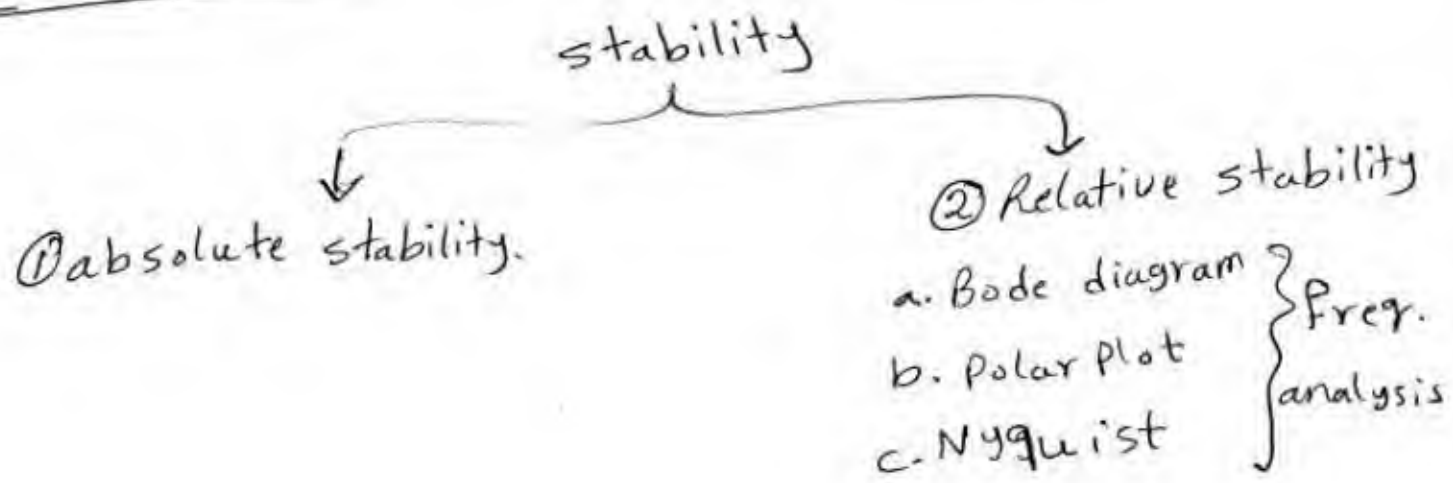


Control Lec 6

Polar Plot

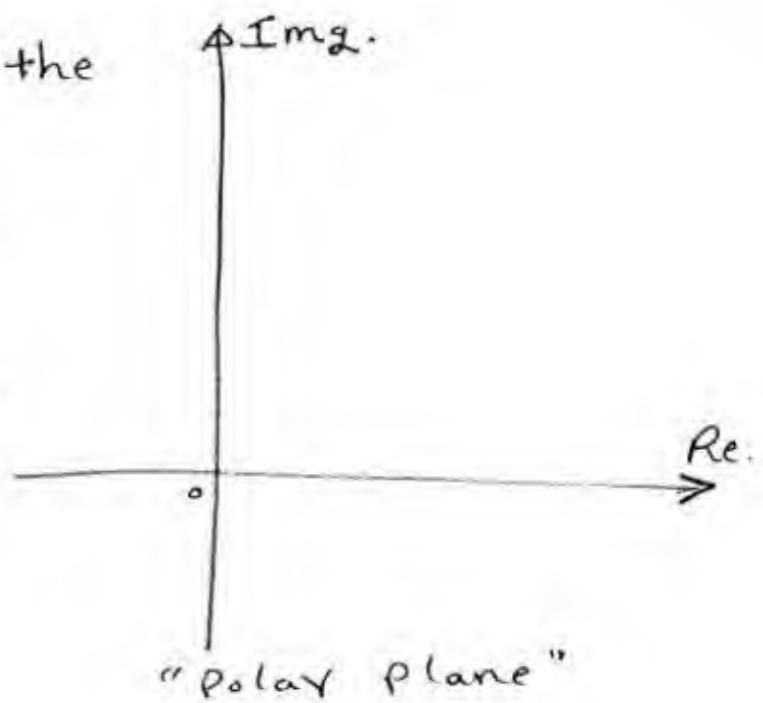


Polar Plot :

→ one of common methods to study Relative stability using freq. analysis technique.

* Polar Plot is a graph for o.l.t.f in polar plane. which consists of two orthogonal axis (Real & Imag.) like s-domain.

* Polar plot represent the $|GH|$ and $\angle GH$ in Polar Plane for different values of ω .



steps to draw it

① Map from s -domain to ω -domain

$$s \rightarrow j\omega \Rightarrow GH(j\omega) = GH(s) \Big|_{s \Rightarrow j\omega}$$

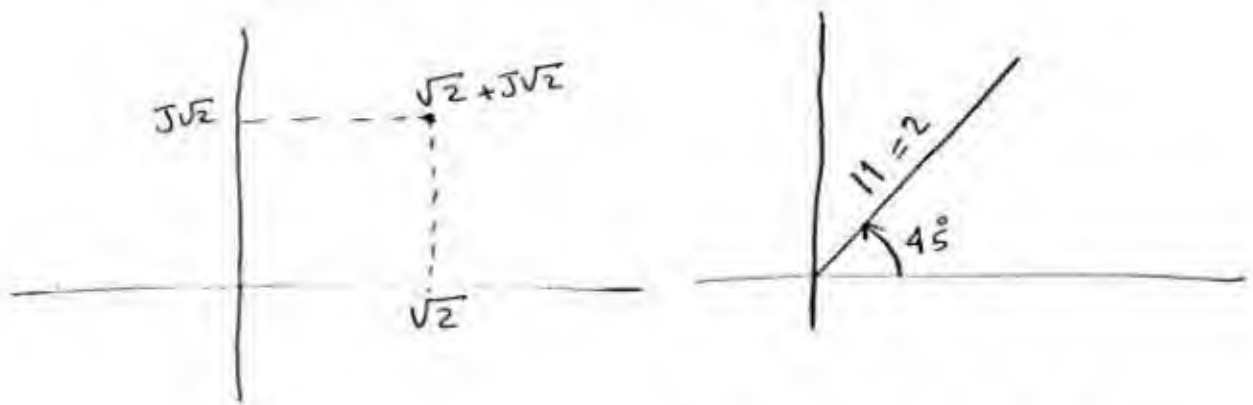
② obtain $|GH(j\omega)|$ and $\phi(\omega) = \angle GH(j\omega)$

③ Draw the $|GH(j\omega)|$ and $\phi(\omega)$

for different values of ω in Polar plane

$$* GH(s) = \sqrt{2} + j\sqrt{2}$$

$$= \sqrt{2+2} \angle \tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = 2 \angle 45^\circ$$



همكه تعبر عنها بالشكلين.

$$\boxed{\text{Ex}} \quad GH(s) = \frac{1}{1+0.5s}$$

→ Draw the polar plot and find PM & GM.

Sol

$$GH(j\omega) = \frac{1}{1+0.5j\omega}$$

$$|GH(j\omega)| = \frac{1}{\sqrt{1+(0.5\omega)^2}} =$$

$$\angle GH(j\omega) = 0 - \tan^{-1}(0.5)\omega = \phi(\omega)$$

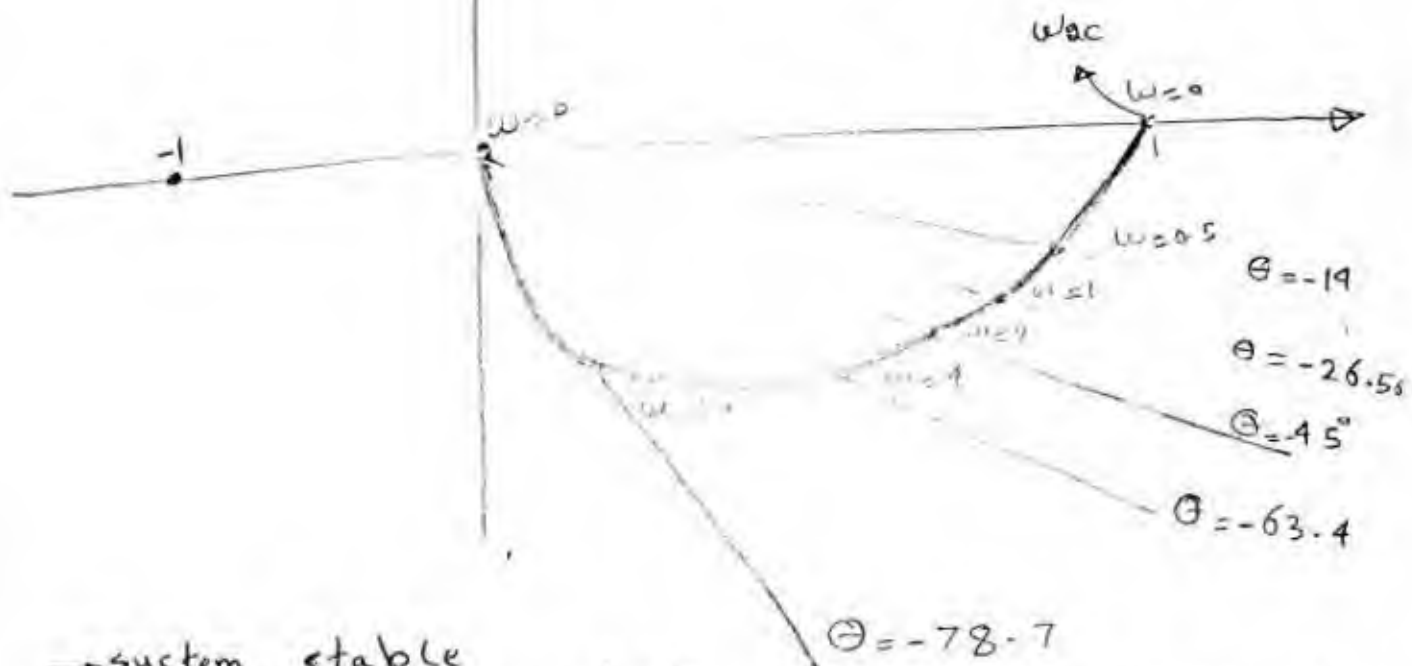
ω	0	0.5	1	2	4	10	∞
$ GH(j\omega) $	1	0.97	0.89	$\frac{1}{\sqrt{2}} \downarrow$ 0.707	0.447	$\frac{1}{\sqrt{26}} = 0.2$	0
$\phi(\omega)$	0	-14.04	-26.56	-45	-63.4	-78.7°	-90

$$\omega_{ac} \Rightarrow \omega \text{ at } |GH(j\omega)| = 1$$

$$L = 0$$

$$PM = 180 + \phi(\omega_{ac})$$

$$= 180^\circ \text{ (true)} \rightarrow \text{stable}$$

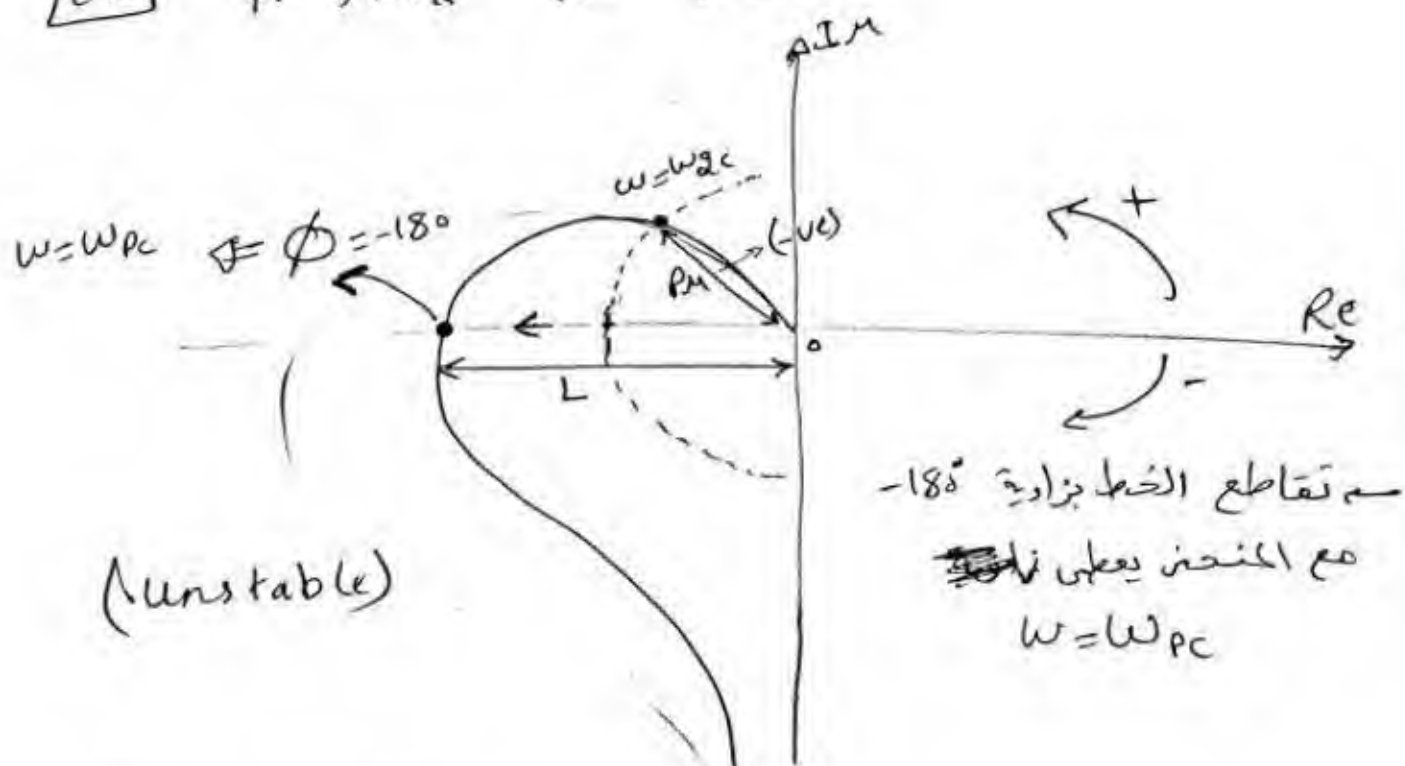


→ system stable

لأنه لم يعتوى (-1) بدخلة
 سيتم توفيرها في النقاط السابقة

GM — غير موجود في المثال ده لأنه لا يوجد
 (ω_{pc}) هنا توولنا لـ -180° لأنه آخر حاجة عندنا -90°

[Ex] to Find PM & GM



$$PM = 180^\circ + \phi(\omega_{gc})$$

$$\downarrow$$

$$|| = 1$$

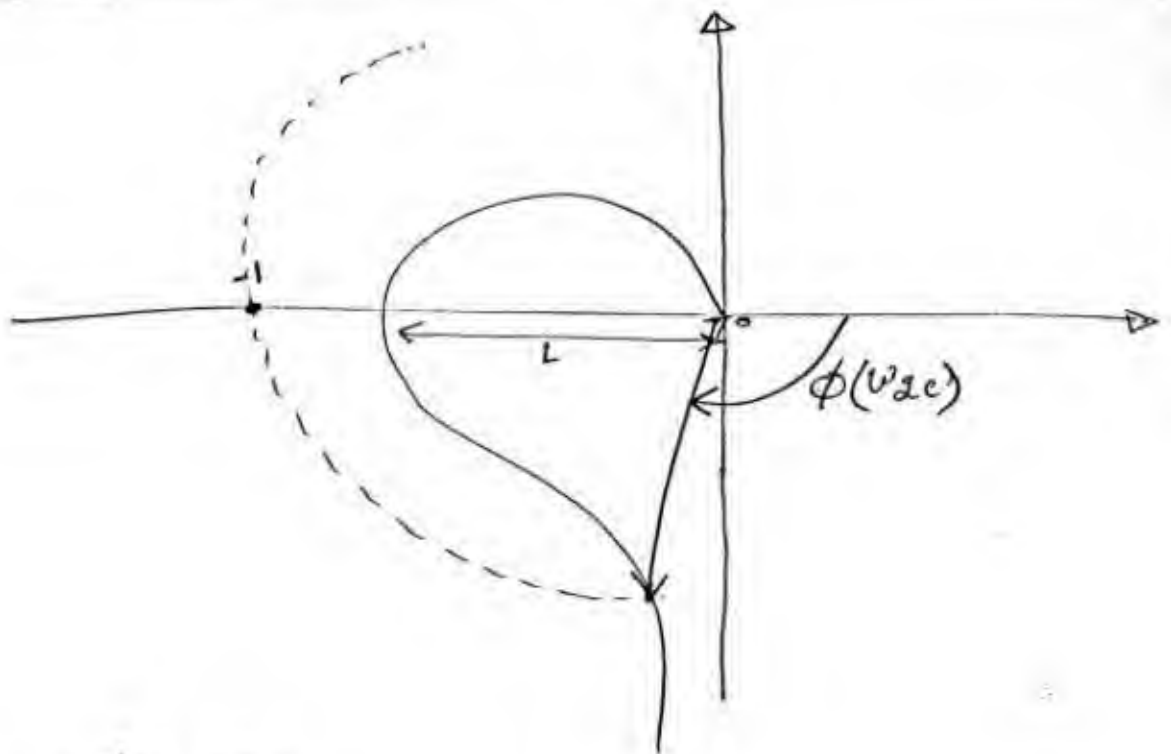
$$||_{dB} = 0$$

مع نرسم نعيد دائرة نصف قطرها = 1 مع عند القطر تتقاطع

مع المنحنى في النقطة $\omega = \omega_{gc}$

ال PM مع هو المسافة من القطر حتى $\omega = \omega_{gc}$

$$GM = \frac{1}{L} \Rightarrow GM|_{dB} = 20 \log\left(\frac{1}{L}\right)$$



$$GM = \frac{1}{L} > 1$$

→ stable

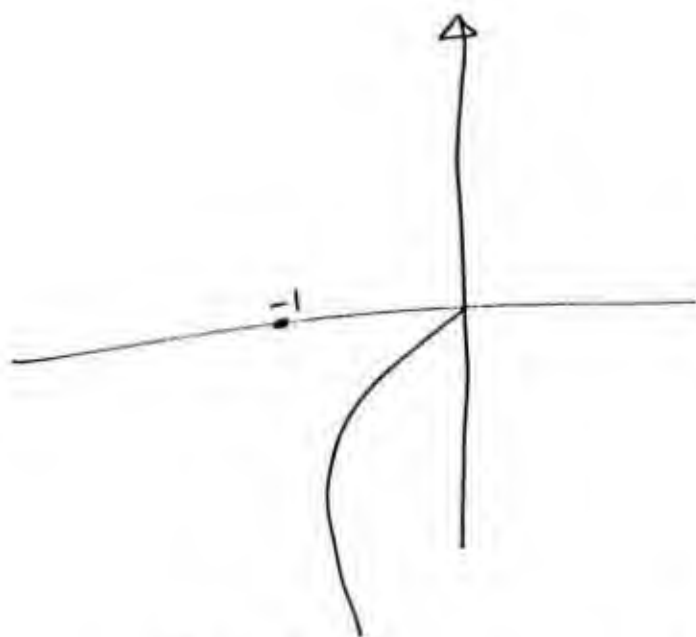
$$PM = 180 + \phi(\omega_{gc})$$

= +ve

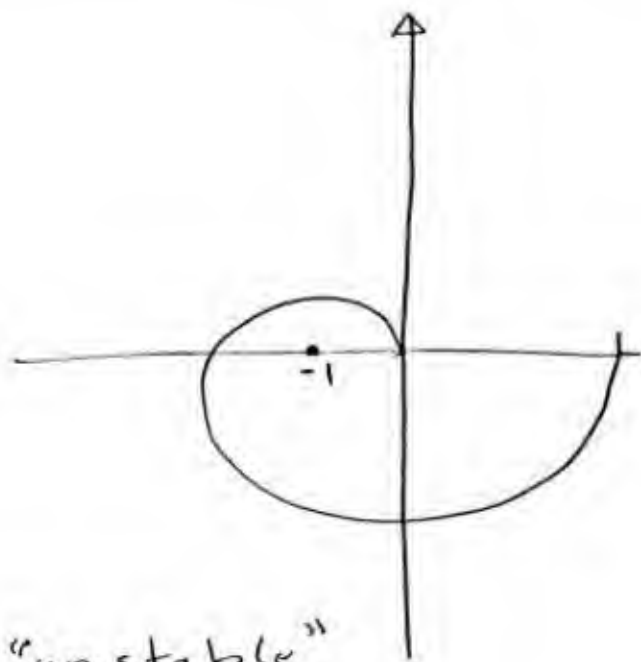
$\phi < -180$

→ system stable

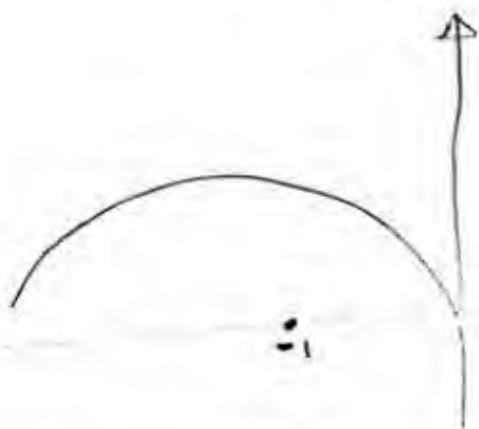
مع هذه الهندسة السابقة والهندسة دي نستنتج ان لو المنحنى احسوى (-1) بداخله يكون (unstable) واذ خرجت 1- بعيد عن المنحنى يكون (stable)



"stable"



"unstable"



→ unstable

$$\boxed{2} \quad G H(s) = \frac{1}{(2s+1)(0.5s+1)(s+1)}$$

$$s \rightarrow j\omega$$

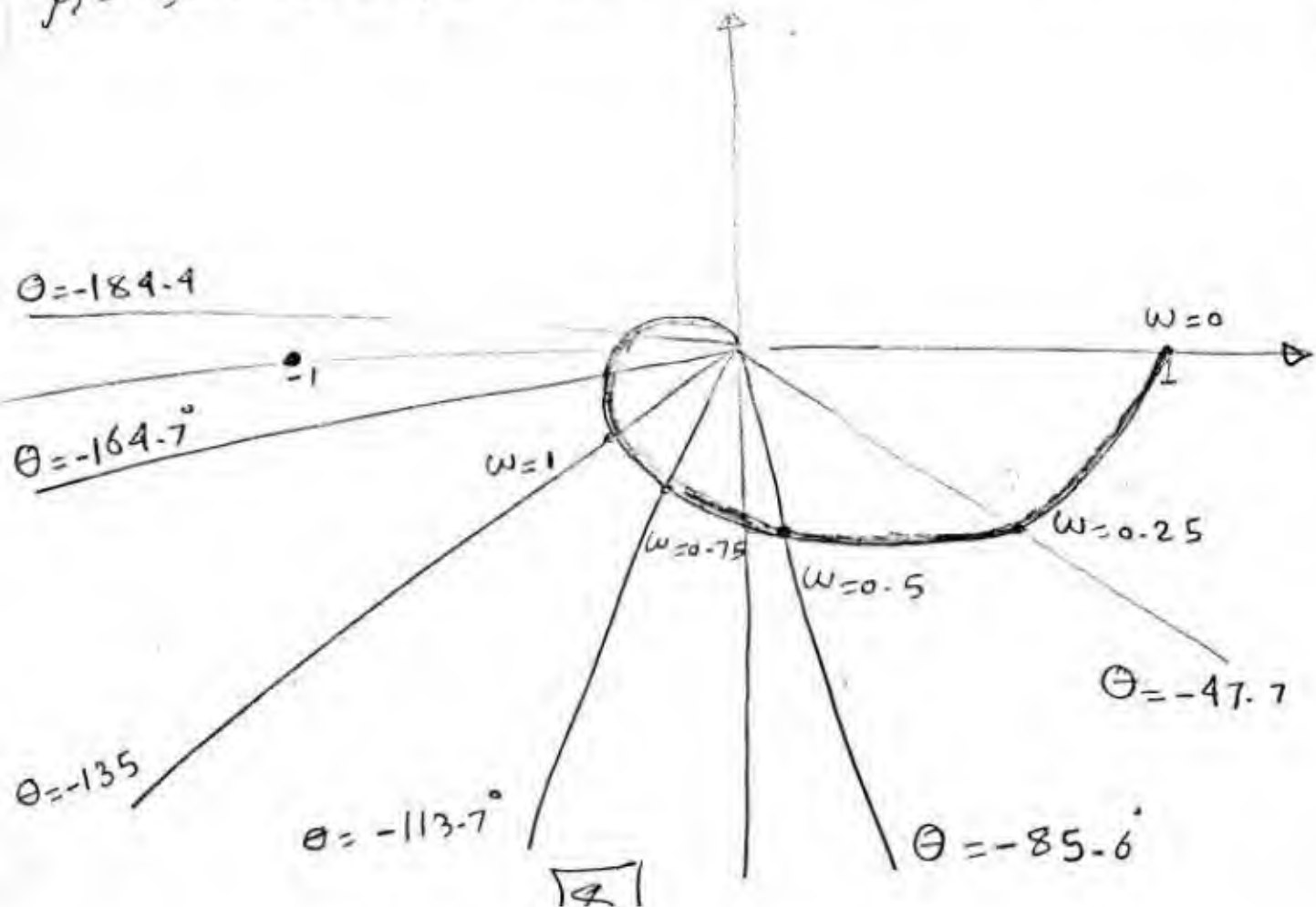
$$G H(j\omega) = \frac{1}{(2j\omega+1)(0.5j\omega+1)(j\omega+1)}$$

$$|GH(j\omega)| = \frac{1}{\sqrt{4\omega^2 + 1} \cdot \sqrt{0.25\omega^2 + 1} \cdot \sqrt{\omega^2 + 1}}$$

$$\phi(\omega) = -\tan^{-1}(2\omega) - \tan^{-1}(0.5\omega) - \tan^{-1}(\omega)$$

ω	0	0.25	0.5	0.75	1	1.5	2	5	∞
$ GH(j\omega) $	1	0.86	0.614	0.415	0.28	0.14	0.077	0.072	0
$\phi(\omega)$	0	-47.7°	-85.6°	-113.7°	-135°	-164.7°	-184.4°	-231.2°	-270°

→ في (Physical sys) ال (Polar Plot) تتغير عند القيمة = 0



→ system stable

$$\underline{\underline{PM}} \quad PM = 180 + \phi(\omega_{gc}) \quad , \quad \omega_{gc} = 0$$

$$= 180 + 0 = 180 \quad (+ve) \rightarrow \text{stable}$$

مع تبحث عن القيمة التي عندها $(1 = |GH(j\omega)|)$ وتكشف
الـ ω عندها value = ω_{gc} ونحسب الزاوية عندها.

GM

$$\omega_{pc} = \omega \quad \text{at} \quad \phi(\omega) = -180^\circ$$

مع حل بسيط — تبهر في الجدول وتكشف أقرب زاوية لـ -180°
وتقلل قيمة الـ ω حتى تصل لـ -180°

حساب تقريبي

* using try and error

$$\omega = 2 \Rightarrow \phi(\omega) = -184.1^\circ$$

$$\omega = 1.9 \Rightarrow \phi(\omega) = -181.029^\circ$$

$$\omega = 1.8 \Rightarrow \phi(\omega) = -177.4^\circ$$

$$\omega = 1.88 \Rightarrow \phi(\omega) = -180.32^\circ$$

$$\omega_{pc} \approx 1.88 \quad \text{rad/sec}$$

$$GM = \frac{1}{|GH(j\omega)|_{\omega=\omega_{pc}=1.88}}$$

$$GM \approx 11.37 > 1 \text{ (stable)}$$

or

$$GM_{dB} = 20 \log GM = 20 \log (11.37) \\ = 21.16 \text{ dB}$$

$$GM \rightarrow +ve \text{ (stable)}$$

سے لو غائر اُجسب ~~ω_{pc}~~ بدل ریاضی

$$\phi(\omega) = -\tan^{-1}(2\omega) - \tan^{-1}(0.5\omega) - \tan^{-1}(\omega)$$

$$\omega_{pc} \rightarrow \omega \text{ at } \phi(\omega) = -180$$

$$-180 = -\tan^{-1}(2\omega) - \tan^{-1}(0.5\omega) - \tan^{-1}(\omega)$$

$$\tan(x \pm y) = \frac{\tan(x) \pm \tan(y)}{1 \mp \tan x \cdot \tan y}$$

$$180 - \tan^{-1}(2\omega_{pc}) = \tan^{-1}(0.5\omega_{pc}) + \tan^{-1}(\omega_{pc})$$

نأخذ tan للطرفين

$$\frac{\tan(180) - 2\omega_{pc}}{1 + \tan(180) \cdot (2\omega_{pc})} = \frac{0.5\omega_{pc} + \omega_{pc}}{1 - 0.5\omega_{pc}^2}$$

$$-2\omega_{pc} = \frac{1.5\omega_{pc} + \cancel{\omega_{pc}}}{1 - 0.5\omega_{pc}^2}$$

$$1.5 = -2 + \omega_{pc}^2 \Rightarrow \omega_{pc}^2 = 3.5$$

$$\omega_{pc} = \sqrt{3.5} = 1.8708 \text{ rad/sec}$$

GM (غير مستقر) ←

$$\boxed{3} \quad G.H(s) = \frac{1}{s(1+s)(0.5+s)}$$

type 1
sys.

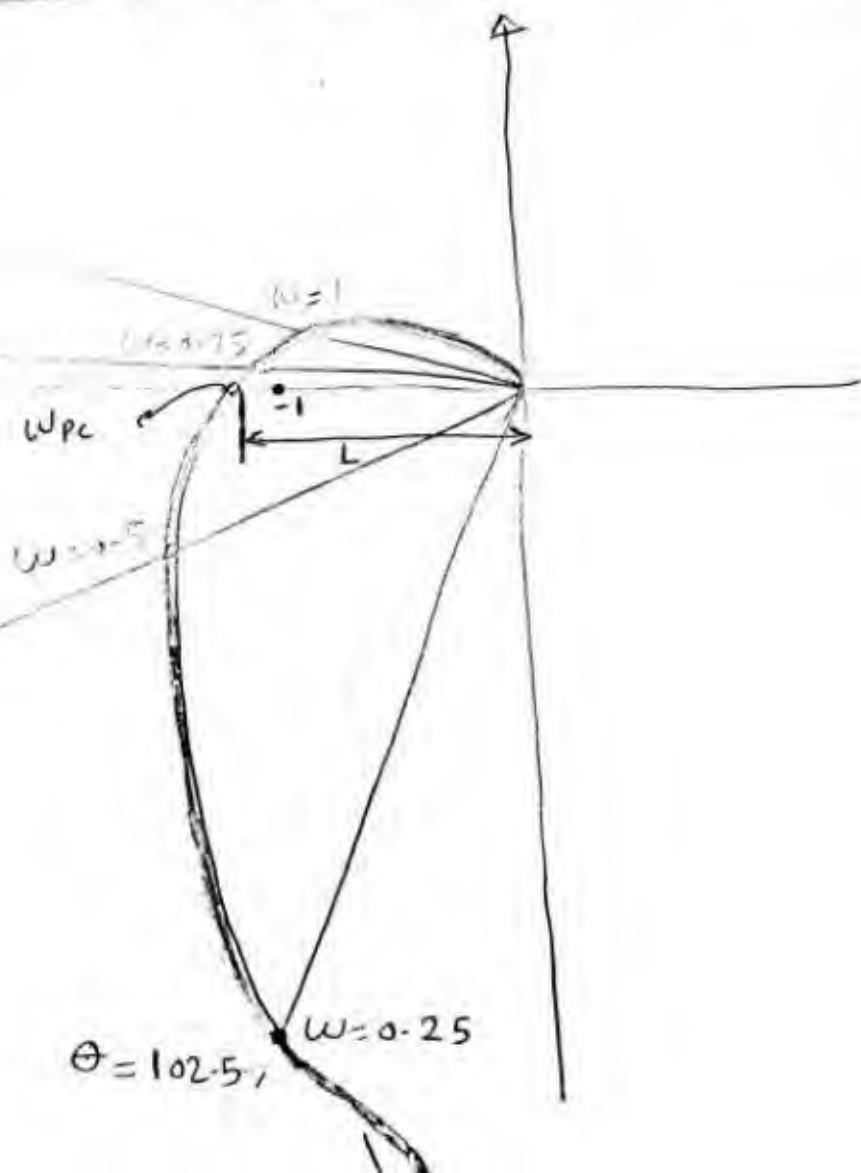
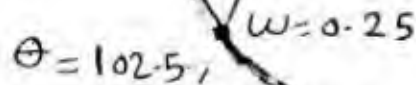
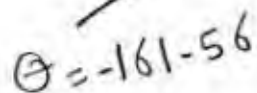
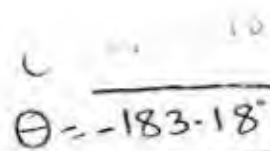
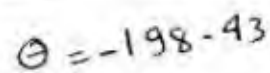
$$s \rightarrow j\omega$$

$$G.H(j\omega) = \frac{1}{j\omega(1+j\omega)(0.5+j\omega)}$$

1

$$\phi(\omega) = -90^\circ - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$$

ω	0	0.25	0.5	0.75	1	1.5	2	∞
$ GH(j\omega) $	∞	6.9	2.53	1.183	0.6	0.234	0.108	0
$\phi(\omega)$	-90°	-102.5°	-161.56°	-183.18°	-198.43°	-217°	-229.4°	-270°



$$GM = \frac{1}{L} < 1 \quad \text{un stable}$$

or

$$\omega_{pc} \Rightarrow \omega \text{ at } \phi = -180^\circ$$

$$\omega = 0.75 \Rightarrow \phi(\omega) = -183.18^\circ$$

$$\omega = 0.7 \Rightarrow \phi(\omega) = -179.4^\circ$$

$$\omega = 0.72 \Rightarrow \phi(\omega) = -180.97^\circ$$

$$\omega = 0.71 \Rightarrow \phi(\omega) = -180.22^\circ$$

$$\omega = 0.708 \Rightarrow \phi(\omega) = -180.068^\circ$$

$$\omega_{pc} \approx 0.708 \text{ rad/sec}$$

$$GM = \frac{1}{|GH|_{\omega=\omega_{pc}}} = 0.752 < 1 \rightarrow \text{un stable}$$

$$\omega_{gc} \Rightarrow \omega \text{ at } |GH| = 1$$

$$\omega = 0.75 \Rightarrow || = 1.183$$

$$\omega = 0.8 \Rightarrow || = 1.034$$

$$\omega = 0.81 \Rightarrow || = 1.0078$$

$$\omega_{gc} \approx 0.81 \text{ rad/sec}$$

$$PM = 18^\circ + \phi (\omega_{gc} = 0.81)$$

$$= -7.32^\circ < 0 \quad (\text{unstable})$$

سے فی ال (mid term) ہیجی سوال لا (Polar Plot)
 مش ہیجی فی الفاینال.

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